Mathematical Lineage: Insoluble Insolvency

Let:

- B_t= stock of debt at time t
- r_t= effective interest rate on debt
- G_t= primary spending
- T_t= revenues
- $S_t = T_t - G_t$= primary surplus
- M_t= seigniorage / inflation finance
- A_t= asset sales or other one-off financing
- R_t= restructuring relief

Then the one-period law of motion is

$$B_{t+1} = (1 + r_t)B_t - S_t - M_t - A_t - R_t$$

Divide by GDP Y_t. Let $b_t = B_t/Y_t$, let nominal GDP growth be g_t, and let lower-case variables denote GDP ratios. Then approximately

$$b_{t+1} = \frac{1 + r_t}{1 + g_t} b_t - s_t - m_t - a_t - \rho_t$$

where $s_t$is the primary surplus ratio, $m_t$the seigniorage ratio, $a_t$asset sales ratio, and $\rho_t$restructuring relief ratio.

2. Condition for mere stabilization

Debt is stable when $b_{t+1} = b_t = b^*$. Solving,

$$s_t + m_t + a_t + \rho_t = \left(\frac{1 + r_t}{1 + g_t} - 1\right) b^*$$

Using the common approximation $\frac{1+r}{1+g} - 1 \approx r - g$,

$$s_t + m_t + a_t + \rho_t \approx (r - g)b^*$$

This is the **required adjustment condition**: the government must generate enough fiscal and quasi-fiscal adjustment to offset the debt-dynamics gap $(r - g)b$.

3. Introduce feasible policy capacity

Now define the **maximum sustainable adjustment capacity** under existing institutions:

$$\Phi_t = \bar{s}_t + \bar{m}_t + \bar{a}_t + \bar{\rho}_t$$

where

- $\bar{s}_t$= maximum politically/administratively feasible primary surplus
- $\bar{m}_t$= maximum tolerable inflation/seigniorage before monetary breakdown
- $\bar{a}_t$= realistic asset-sale proceeds
- $\bar{\rho}_t$= feasible restructuring relief

CRE's own framing is close to this idea: "insoluble insolvency" exists when the feasible set of policy actions cannot restore solvency under existing constraints.

4. Define solvency gap

Define the **required adjustment**:

$$\Psi_t = (r_t - g_t)b_t$$

and the **solvency gap**:

$$\Delta_t = \Psi_t - \Phi_t$$

Then:

- If $\Delta_t \leq 0$, solvency is difficult but still feasible.
- If $\Delta_t > 0$, ordinary remedies are insufficient.

This yields the formal condition for **insoluble insolvency**:

$$(r_t - g_t)b_t > \bar{s}_t + \bar{m}_t + \bar{a}_t + \bar{\rho}_t$$

That is the core derivation.

5. Add future obligations, not just current debt

For sovereigns, current debt alone is too narrow. Let

$$\Omega_t = B_t + PV_t(\text{unfunded commitments})$$

where $PV_t(\cdot)$is the present value of implicit and explicit future obligations. Then the broader solvency ratio is

$$\omega_t = \frac{\Omega_t}{Y_t}$$

Now the required adjustment becomes

$$\Psi_t^* = (r_t - g_t)\omega_t$$

and the insoluble-insolvency condition becomes

$$(r_t - g_t)\omega_t > \Phi_t$$

This is the more powerful version, because it captures the intuition that a government may appear liquid on current debt service while being structurally insolvent once entitlement and rollover commitments are included. That is also consistent with CRE's description tying the concept to unfunded liabilities, demographic pressures, structural deficits, and political inability to reform.

6. Make the "insoluble" part explicit

Ordinary insolvency means liabilities exceed resources absent adjustment.

Insoluble insolvency means the adjustment needed exceeds the adjustment capacity:

$$\underbrace{\text{Needed correction}}_{\Psi_t^*} > \underbrace{\text{Maximum feasible correction}}_{\Phi_t}$$

Equivalently, define the feasibility ratio

$$\kappa_t = \frac{\Phi_t}{\Psi_t^*}$$

Then

- $\kappa_t \geq 1$: solvency is theoretically restorable
- $\kappa_t < 1$: insolvency is **insoluble** under current institutional constraints

So the condition can be written very simply as

$$\kappa_t < 1$$

7. Dynamic version

A sovereign enters an insoluble state when the gap is persistent, not merely temporary. So require

$$\Delta_t > 0 \text{for all } t \in [T, T+n]$$

for some nontrivial horizon n, and especially if

$$E_t[\Delta_{t+j}] > 0 \forall j \geq 0$$

Then the government is not just illiquid this year; it is trapped in a regime where the feasible policy frontier never catches the required adjustment.

8. Why growth alone may not solve it

A common objection is: “faster growth solves debt.” But in this framework growth helps only by lowering $(r-g)$. If obligations also rise with aging, health costs, and interest compounding, then the relevant stock $\omega_t$may grow faster than GDP:

$$\dot{\omega}_t > 0$$

even if $g_t$rises modestly. Thus growth is insufficient when

$$(r_t - g_t)\omega_t$$

remains above feasible adjustment capacity despite optimistic growth assumptions.

9. Final formal definition

A sovereign is in a state of **insoluble insolvency** at time $t$iff

$$\boxed{(r_t - g_t)\omega_t > \bar{s}_t + \bar{m}_t + \bar{a}_t + \bar{\rho}_t}$$

with $\omega_t$including both funded debt and the present value of unavoidable future obligations.

In words:

Insoluble insolvency exists when the debt-stabilizing adjustment required by the sovereign's total obligations exceeds the maximum adjustment that can be achieved through taxation, spending restraint, inflation finance, asset sales, or restructuring under existing political, legal, administrative, and macroeconomic constraints. This matches the public CRE framing that ordinary remedies can no longer restore solvency.

10. One-line intuition

Ordinary insolvency:

$$\text{resources} < \text{obligations}$$

Insoluble insolvency:

$$\text{maximum possible correction} < \text{correction required}$$